Robotics

Presented by Tom Gray

Robotics Definition

- Robotics, the combination of motion control and remote manipulation

Robotics

- Robotics I: Architectures
- Robotics II: Kinematics
- Robotics III: Inverse Kinematics
- Robotics IV: Feedback

Robotics I

Architectures

Architecture – Cartesian

- Direct Control for X, Y, Z and θ
- Stiff Structure
- Large Workspace

Architecture – Articulated Arm

- Most recognizable robot.
- Compact when not in use
- Can enter a workspace for various angles

Architecture - SCARA

- High Speed motion in a plane
- Extremely precise
- Low carrying capacity

Architecture – Stewart Platform

- Very Stiff Base
- Small workspace
- 6 DOF
- Complex positioning mathematics
- Great Disneyland rides

Robotics II

Kinematics

Gantry Kinematics

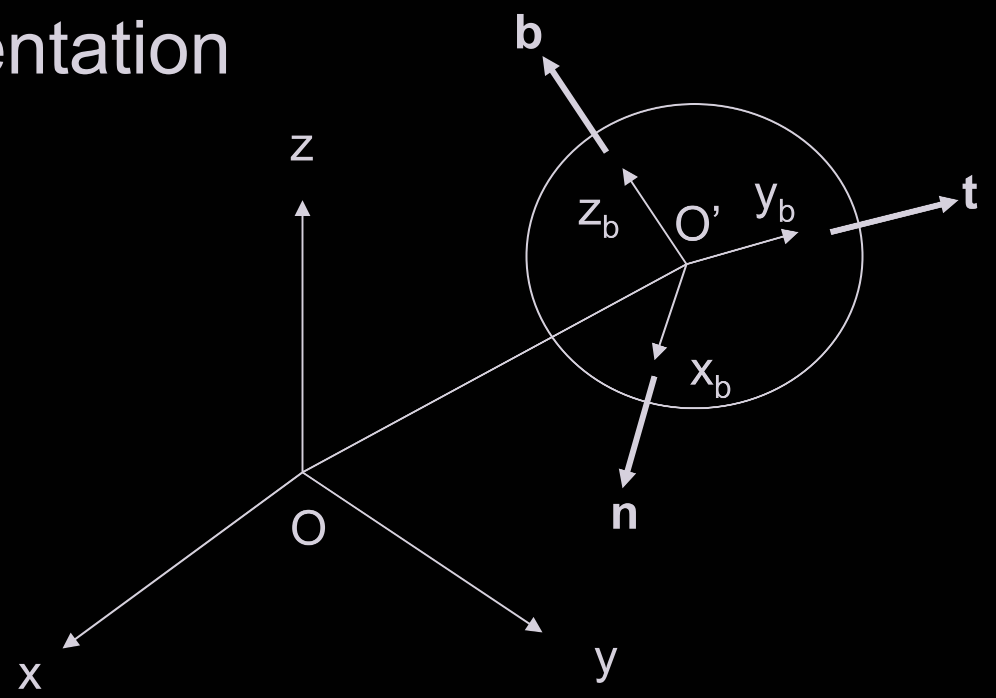
- To move the robot 10 inches in the x direction, move the x axis 10 inches in the x direction.
- Can anyone tell me how to move in the y direction?

Articulated Arm Kinematics

- Model the physical relationship between joint positions and end-effector position.

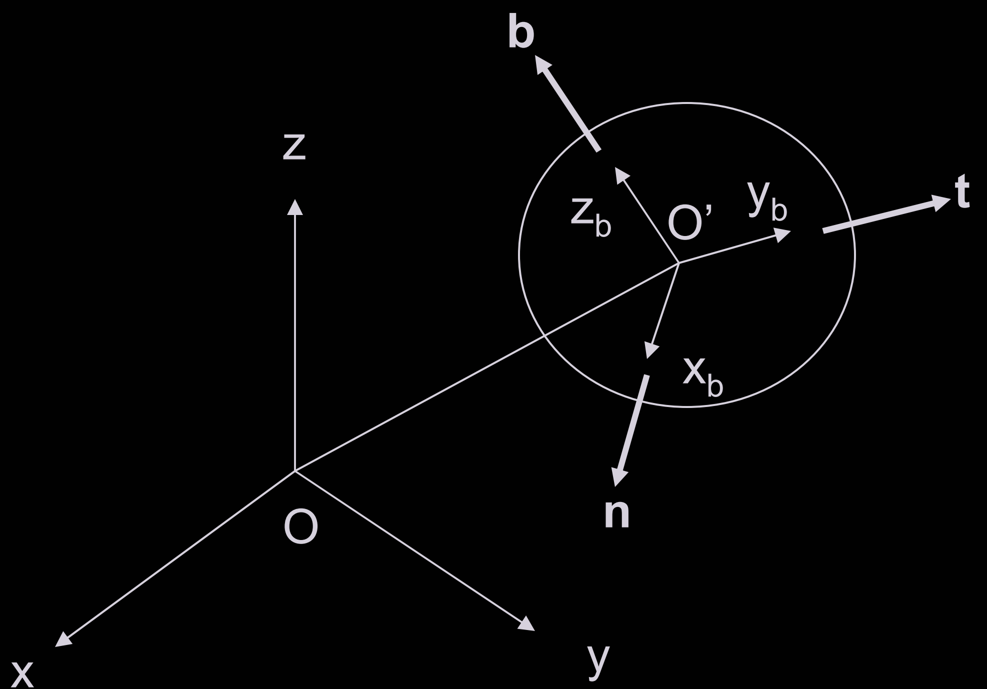
Position and Orientation

- Arm linkages can be modeled as rigid bodies.
- Each body is defined by its position and orientation



Define Position

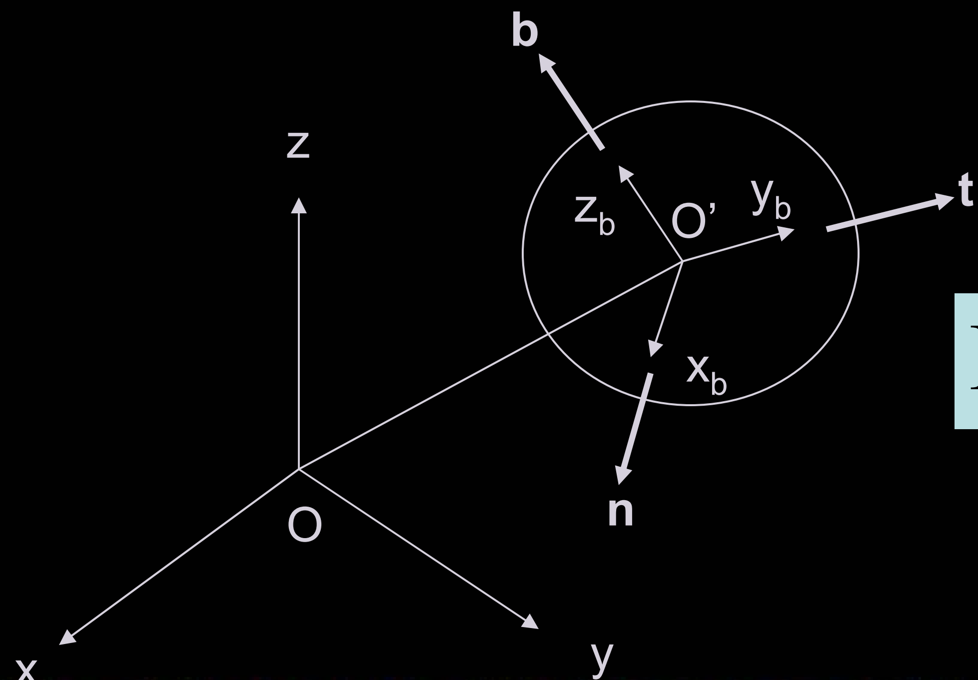
- The position can be defined from an arbitrary coordinate system



$$\mathbf{x}_o = \begin{pmatrix} x_0 \\ y_o \\ z_o \end{pmatrix}$$

Define Orientation

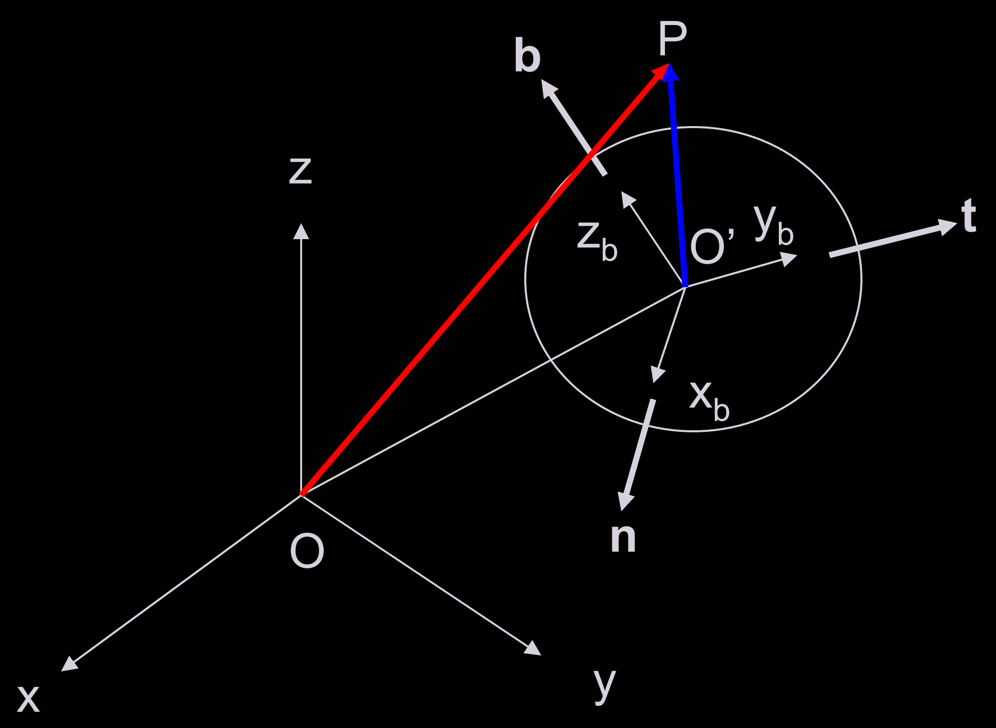
- Three unit vectors n, t and b pointing toward x_b, y_b and z_b.
- R is a 3x3 matrix



$$R = [\mathbf{n}, \mathbf{t}, \mathbf{b}]$$

Coordinate Transformation

- Select and arbitrary point P



Coordinate Transformation

- Point P from frame 0-xyz

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

- Point P from frame 0'-$x_b y_b z_b$

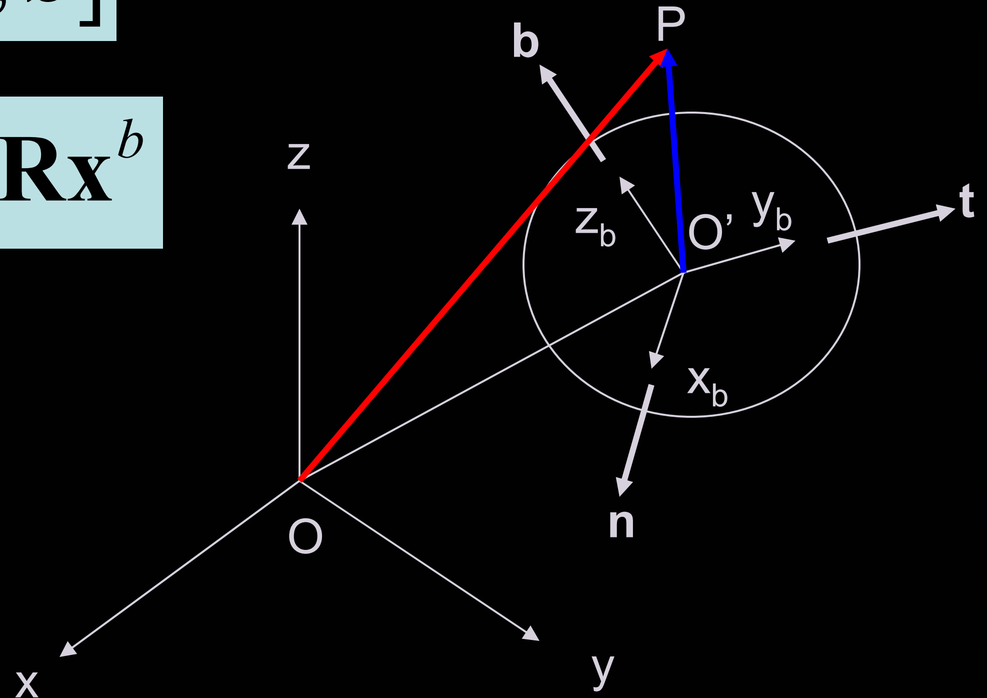
$$\mathbf{x}^b = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Coordinate Transformation

$$\mathbf{x} = \mathbf{x}_o + u\mathbf{n} + v\mathbf{t} + w\mathbf{b}$$

$$\mathbf{R} = [\mathbf{n}, \mathbf{t}, \mathbf{b}]$$

$$\mathbf{x} = \mathbf{x}_o + \mathbf{R}\mathbf{x}^b$$



Inverse Transformation

Forward Transformation

$$\mathbf{x} = \mathbf{x}_o + \mathbf{R}\mathbf{x}^b$$

Multiply by the Transpose of R

$$\mathbf{R}^T\mathbf{x} = \mathbf{R}^T\mathbf{x}_o + \mathbf{R}^T\mathbf{R}\mathbf{x}^b$$

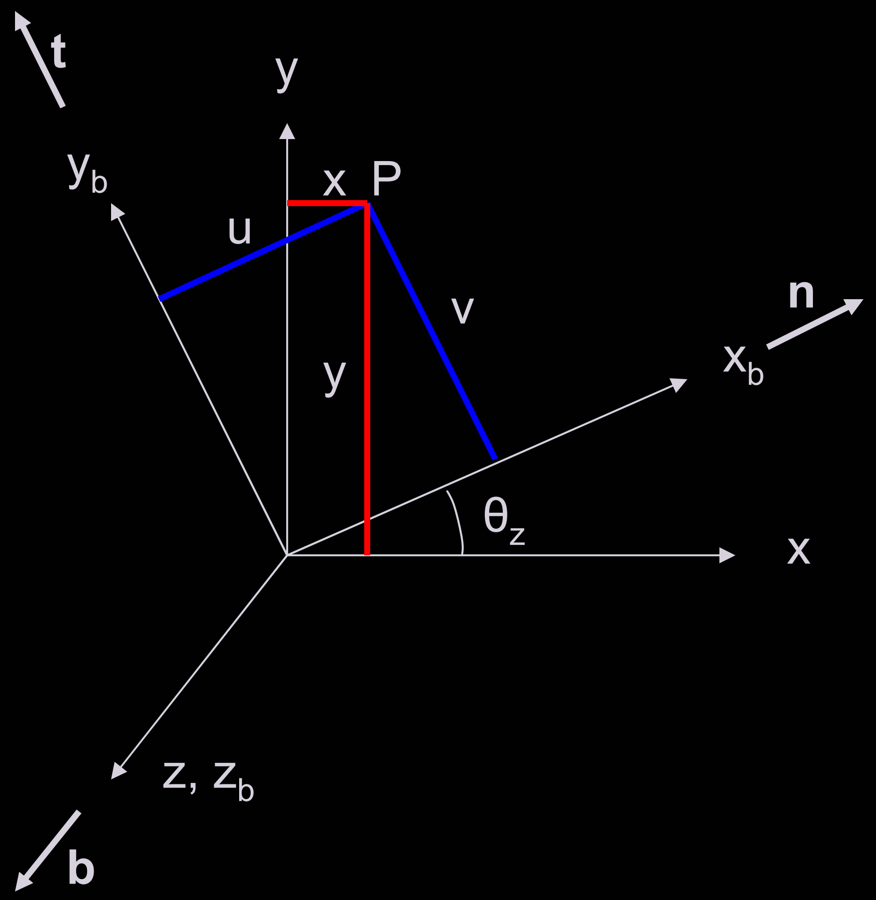
Because n, t and b are orthogonal

$$\mathbf{R}^T\mathbf{R} = \begin{bmatrix} \mathbf{n}^T\mathbf{n} & \mathbf{n}^T\mathbf{t} & \mathbf{n}^T\mathbf{b} \\ \mathbf{t}^T\mathbf{n} & \mathbf{t}^T\mathbf{t} & \mathbf{t}^T\mathbf{b} \\ \mathbf{b}^T\mathbf{n} & \mathbf{b}^T\mathbf{t} & \mathbf{b}^T\mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse Transformation

$$\mathbf{x}^b = -\mathbf{R}^T\mathbf{x}_o + \mathbf{R}^T\mathbf{x}$$

Example: Define n, t, b

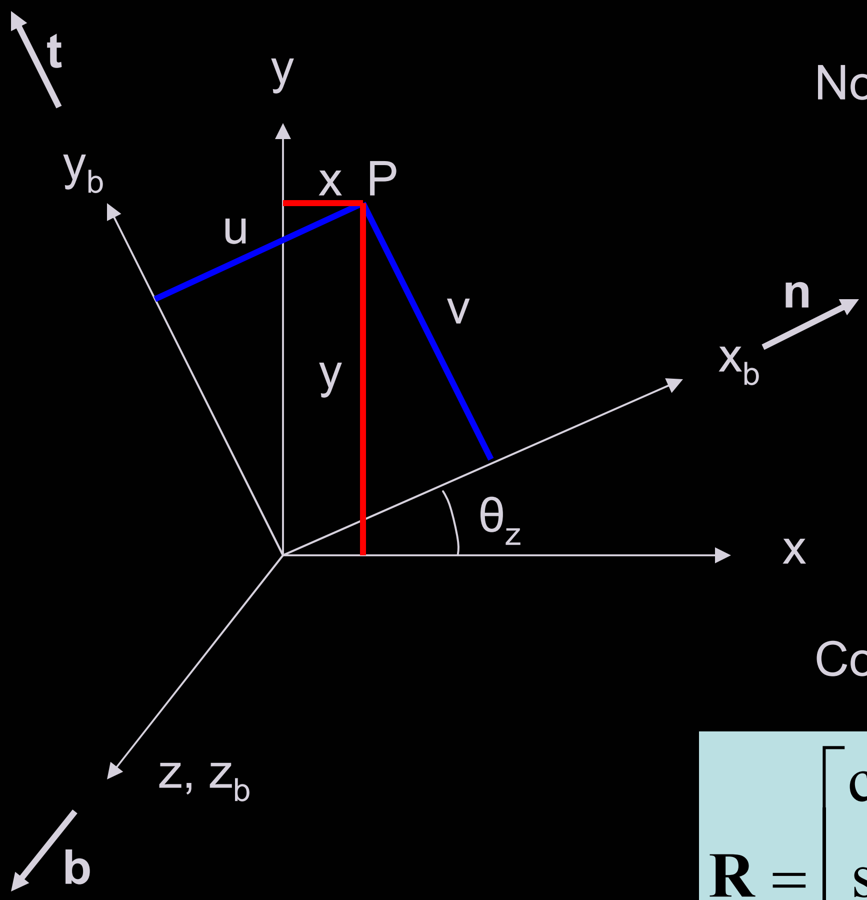


$$\mathbf{n} = \begin{pmatrix} \cos\theta_z \\ \sin\theta_z \\ 0 \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} -\sin\theta_z \\ \cos\theta_z \\ 0 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Example: Define R and x_o



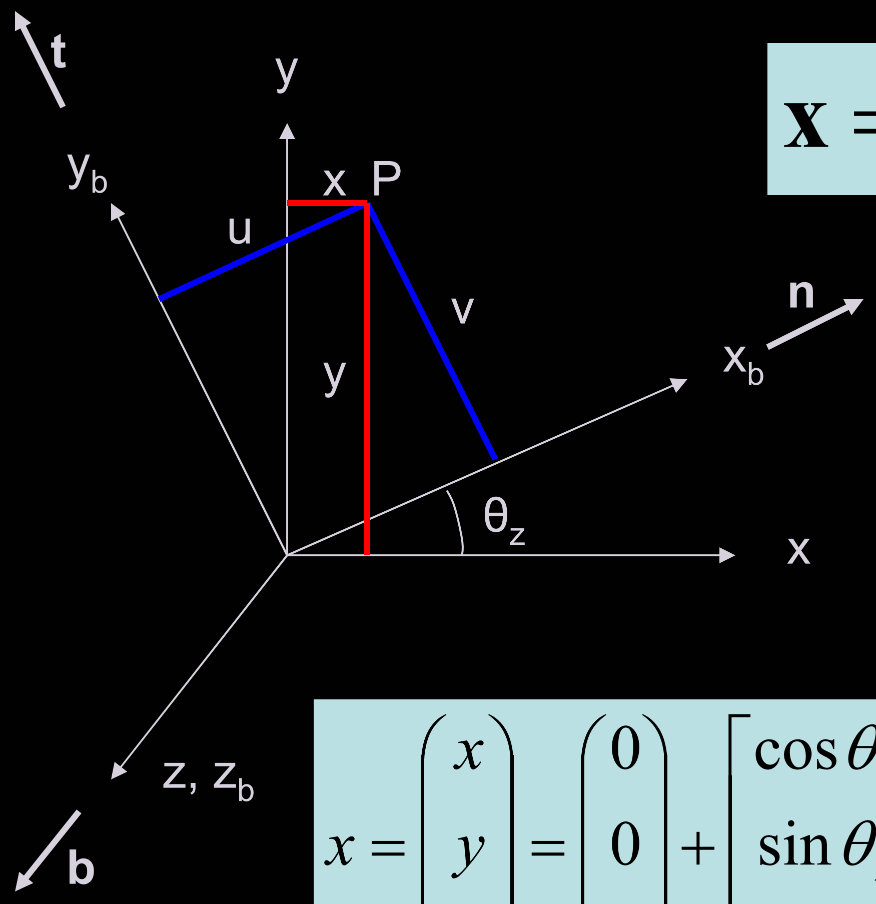
No Translation Means:

$$\mathbf{x}_o = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Combining n, t and b

$$\mathbf{R} = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example: Substitute



$$\mathbf{x} = \mathbf{x}_o + \mathbf{R}\mathbf{x}^b$$

$$x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Rotation Matrix

- R is called the rotation matrix
- R represents the rotation from

  O-xyz to O'-$x_b y_b z_b$

Homogeneous Transformation

- A shorthand notation to combine the translation and rotation matrices

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \qquad \mathbf{X}^b = \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix} \qquad \mathbf{A} = \begin{bmatrix} \begin{bmatrix} & & \\ & \mathbf{R} & \\ & & \end{bmatrix} & \begin{pmatrix} \\ \mathbf{x}_o \\ \\ \end{pmatrix} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

Concise Notation

- Multiple Transformations with standard notation

$$x = x_o + Rx_o^{'} + RR^{'}x^c$$

- Multiple Transformations with homogeneous transformation

$$X^0 = A_1^0 A_2^1 \ldots A_n^{n-1} X^n$$

Inverse Transformation

- Derive the inverse by comparing

$$\mathbf{x}^b = -\mathbf{R}^T\mathbf{x}_o + \mathbf{R}^T\mathbf{x}$$

- And homogeneous transformation

$$\mathbf{X}^b = \mathbf{A}^{-1}\mathbf{X}$$

- To get

$$\mathbf{A}^{-1} = \begin{bmatrix} \begin{bmatrix} & \mathbf{R}^T & \end{bmatrix} & \begin{pmatrix} -\mathbf{R}^T\mathbf{x}_o \end{pmatrix} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

Modeling Manipulators

- Apply this theory to modeling manipulator arms
- Modeling of open kinematic chains
- First link is 0, last link is n
- Attach O_n-$x_n y_n z_n$ to the end effector
- Attach O_i-$x_i y_i z_i$ to each link
- Attach O_0-$x_0 y_0 z_0$ to the base

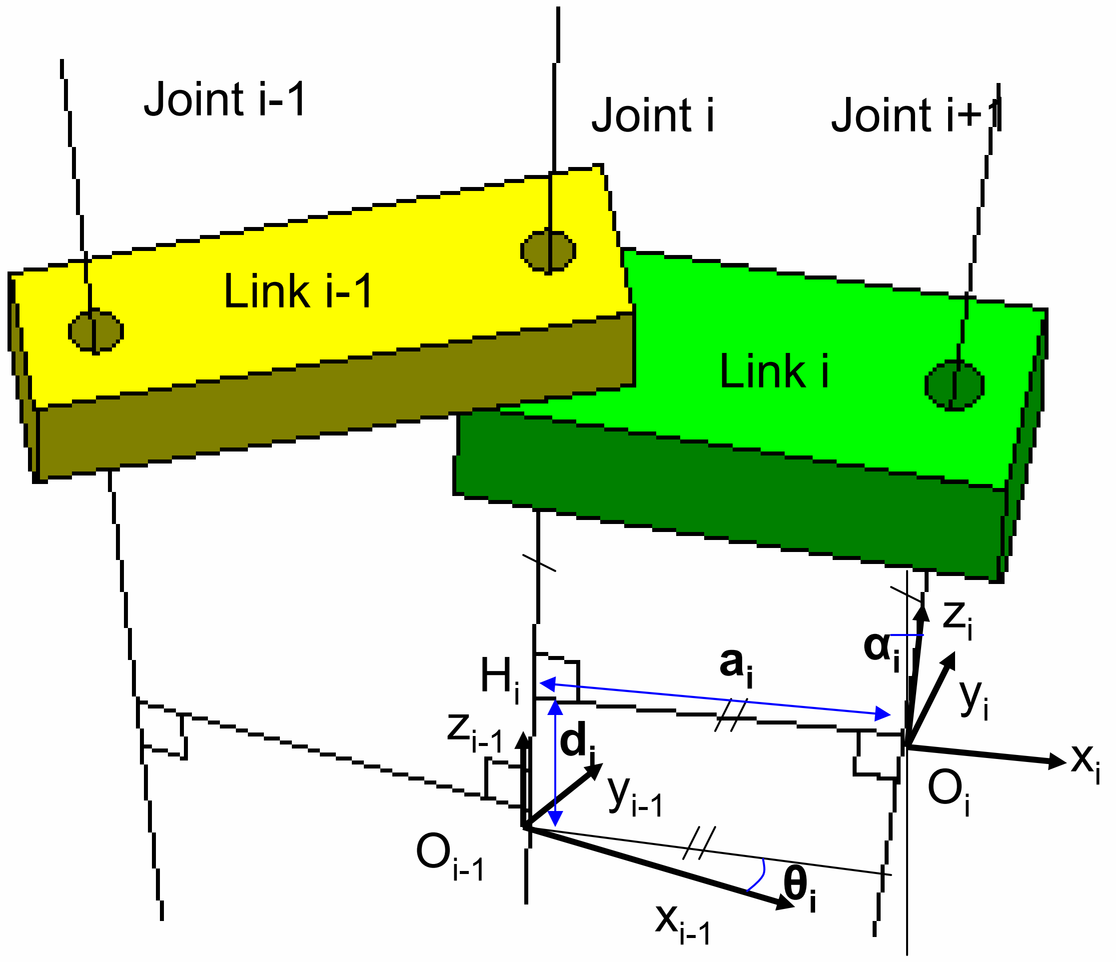
Denavit-Hartenberg Notation

- Uses a minimum number of parameters to describe link geometry
- Describes links consistently

Denavit-Hartenberg Rules

- The origin of the i^{th} coordinate frame is located at the intersection of joint axis i+1 and the common normal between joint axis i and i+1.
- The x_i axis is directed along the common normal
- The z_i axis is along the joint axis i+1
- The y_i axis is chosen to create a right-handed coordinate system

DH Example



Joint i-1
Joint i
Joint i+1
Link i-1
Link i
H_i
a_i
α_i
z_i
y_i
x_i
O_i
z_i-1
d_i
y_i-1
O_i-1
θ_i
x_i-1

DH Definitions

- a_i – the length of the common normal
- d_i – the distance between the origin O_{i-1} and the point H_i
- α_i – the angle between the joint axis i and the z_i axis in the right-hand sense
- θ_i – the angle between the x_{i-1} axis and the common normal H_iO_i measured about the z_{i-1} axis in the right-hand sense

DH Parmeters

- Link Parameters
  - a_i and α_i are constants that represent the length of the link and the twist angle between the joints
- Joint Parameters
  - d_i and θ_i are vary as the joint moves
  - For prismatic joints, only d_i varies
  - For revolute joints, only θ_i varies

The Joint Relationship

- For each joint there is an equation

$$\mathbf{X}^{i-1} = \mathbf{A}_i^{i-1}\mathbf{X}^i$$

- Where

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kinematic Equation

- Since joints can only be prismatic or revolute we use q_i to denote d_i & θ_i
- T is a 4x4 matrix representing the position of the end effector with reference to the base frame

$$\mathbf{T} = \mathbf{A}_1^0(q_1)\mathbf{A}_2^1(q_2)\ldots\mathbf{A}_n^{n-1}(q_n)$$

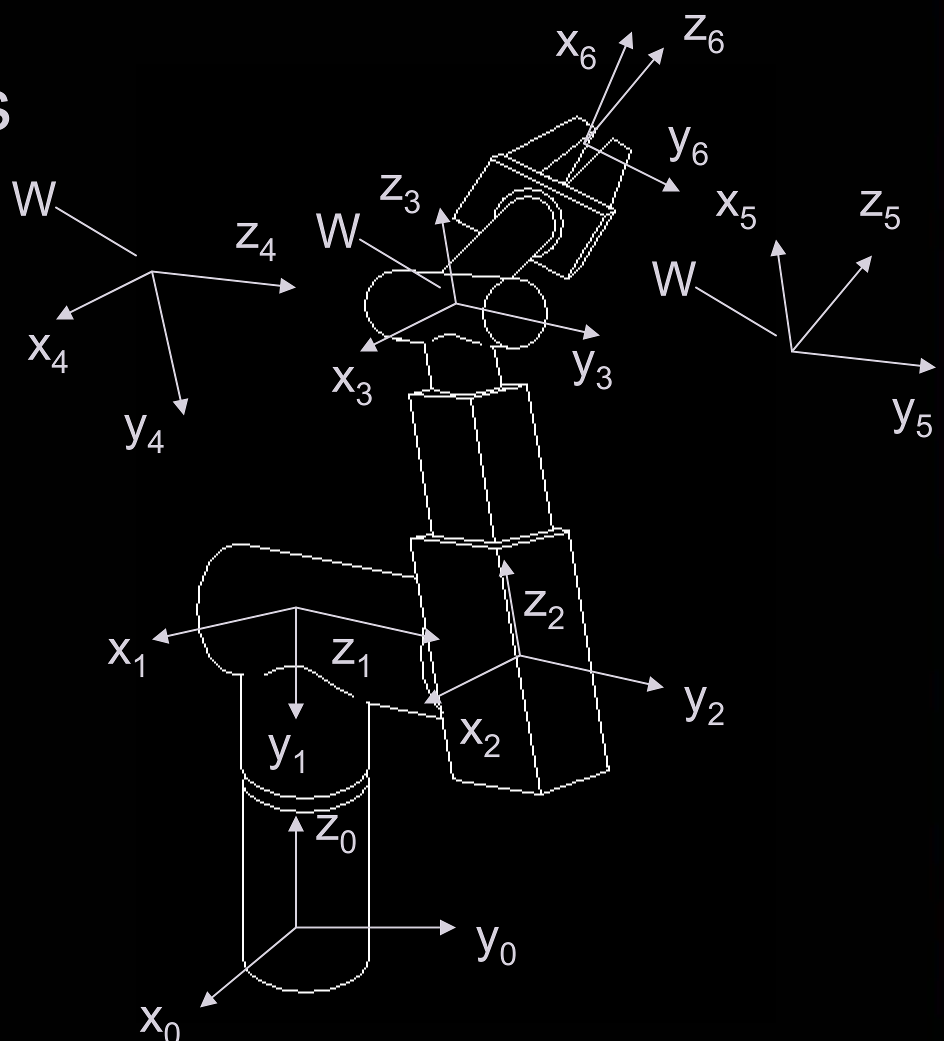
- This is the kinematic equation

Special DH Rules

- End Effector Link
  - Coordinate frame can be locate conveniently.
  - x_n must intersect the last joint axis at a right angle
- Base Link
  - Origin can be located conveniently along joint axis 1
  - z_0 must be parallel to joint axis 1

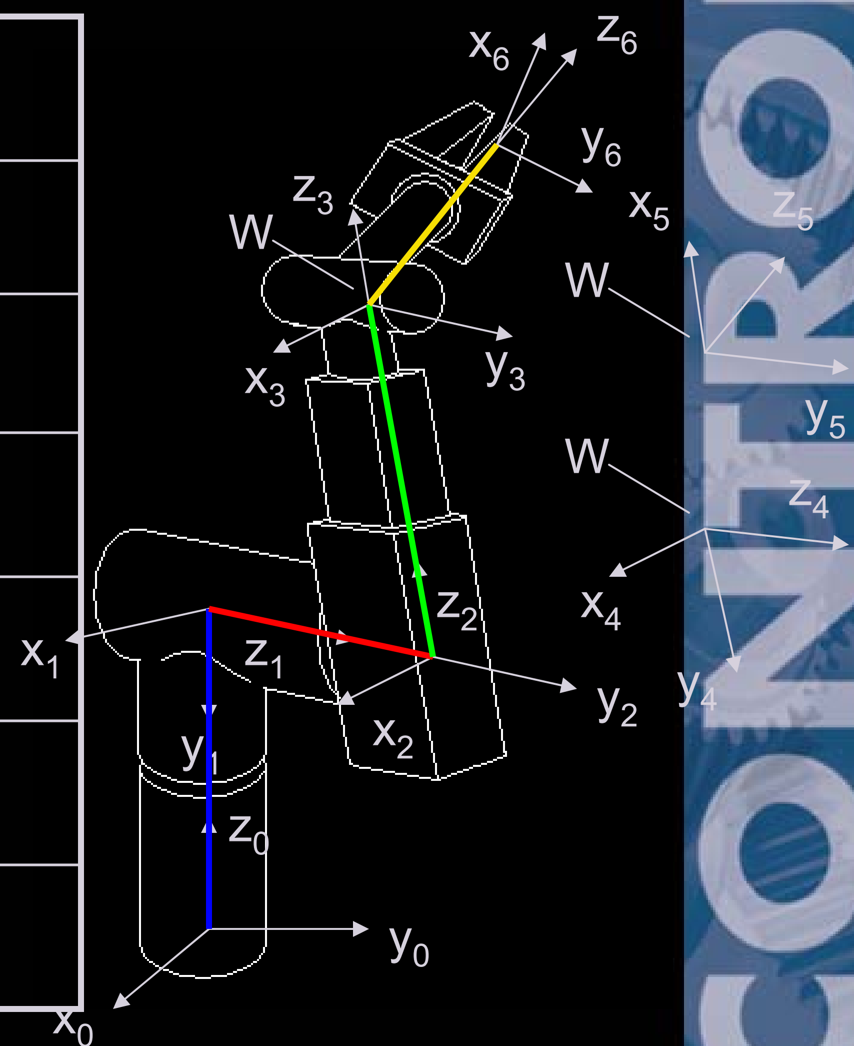
5-R-1-P Example

- Identify Joints
- Place O_0
- Place O_1
- Place O_2
- Place O_3
- Place O_4
- Place O_5



5-R-1-P Example

	a_i	α_i	d_i	θ_i
1	-90	0	l_0	θ_1
2	90	0	l_1	θ_2
3	0	0	d_3	0
4	-90	0	0	θ_4
5	90	0	0	θ_5
6	0	0	l_2	θ_6



5-R-P-1 Example

$$\mathbf{A}_1^0(\theta_1)=\begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_4^3(\theta_4)=\begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_2^1(\theta_2)=\begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_5^4(\theta_5)=\begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_3^2(d_3)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_6^5(\theta_6)=\begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5-R-P-1 Example

- Finally the Kinematic Equation is given by

$$\mathbf{T} = \mathbf{A}_1^0(\theta_1)\mathbf{A}_2^1(\theta_2)\mathbf{A}_3^2(d_3)\mathbf{A}_4^3(\theta_4)\mathbf{A}_5^4(\theta_5)\mathbf{A}_6^5(\theta_6)$$

- Now we can move on to inverse kinematics

Robotics II

Inverse Kinematics

Inverse Kinematics

- Find the angles that provide a specific end effector position and orientation
- N equations and n unknowns

Solvability

- Given a 6 axis robot
- If three axis intersect at a point
- The equations can be divided into sets of 3 simultaneous equations
- It can be proven that there exists a closed form solution for divided simultaneous equations with three unknowns.

R-5-P-1 Example

$$p_x^* = p_x - l_2 b_x \qquad p_y^* = p_y - l_2 b_y \qquad p_z^* = p_z - l_2 b_z$$

$$\theta_1 = 2\tan^{-1}\left[\frac{-p_z^* \pm \sqrt{p_z^{*2} + p_y^{*2} + l_1^2}}{l_1 + p_y^*}\right]$$

$$\theta_2 = \tan^{-1}\left[\frac{p_z^* c_1 + p_y^* s_1}{p_z^* - l_0}\right]$$

$$d_3 = \pm\sqrt{\left(p_z^* c_1 + p_y^* s_1\right)^2 + \left(p_z^* - l_0\right)^2}$$

R-5-P-1 Example

$$\theta_4 = \tan^{-1}\left(\frac{b_{y'}}{b_{z'}}\right)$$

$$\theta_5 = \tan^{-1}\left(\frac{b_{x'}c_4 + b_{y'}s_4}{b_{z'}}\right)$$

$$\theta_6 = \tan^{-1}\left(\frac{-n_{x'}s_4 + n_{y'}c_4}{-t_{x'}s_4 + t_{y'}c_4}\right)$$

Robotics IV

Feedback

Feedback Challenges

- Robots can do remote tasks if communications can keep up.
- Slow connections or long distances limit human operators

Feedback Solutions

- Extra Intelligence and Sensors at the end-effector reduces the communications bandwidth
- Laser Gauges, Radar, Sonar, Vision, Touch probes can help operators do things previously impossible with their own skills.

Thank You

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